MAC 1105
take home worksheet

DO: 15-30, 57-64, 65-72 odds

PROBLEM SET 5.3

Write each of the following in logarithmic form. For example, $2^4 = 16$ becomes $\log_2 16 = 4$.

1. $3^2 = 9$
2. $2^5 = 32$
3. $5^3 = 125$
4. $10^1 = 10$
5. $2^{-4} = \frac{1}{16}$
6. $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$
7. $10^{-2} = 0.01$
8. $10^5 = 100{,}000$

Write each of the following in exponential form. For example, $\log_2 8 = 3$ becomes $2^3 = 8$.

9. $\log_2 64 = 6$
10. $\log_3 27 = 3$
11. $\log_{10} 0.1 = -1$
12. $\log_5\left(\frac{1}{25}\right) = -2$
13. $\log_2\left(\frac{1}{16}\right) = -4$
14. $\log_{10} 0.00001 = -5$

Evaluate each of the following.

15. $\log_6 36$
16. $\log_3 243$
17. $\log_5\left(\frac{1}{5}\right)$
18. $\log_4\left(\frac{1}{64}\right)$
19. $\log_{10} 10$
20. $\log_{10} 1$
21. $\log_3\sqrt{3}$
22. $\log_5\sqrt[3]{25}$
23. $\log_3\left(\frac{\sqrt{27}}{3}\right)$
24. $\log_{1/2}\left(\frac{\sqrt[4]{8}}{2}\right)$
25. $\log_{1/4}\left(\frac{\sqrt[4]{32}}{2}\right)$
26. $\log_2\left(\frac{\sqrt[3]{16}}{4}\right)$
27. $10^{\log_{10} 7}$
28. $5^{\log_5 13}$
29. $\log_2(\log_5 5)$
30. $\log_6(\log_2 64)$

Solve each of the following equations.

31. $\log_5 x = 2$
32. $\log_{10} x = 3$
33. $\log_8 t = \frac{5}{3}$
34. $\log_4 m = \frac{3}{2}$
35. $\log_b 3 = \frac{1}{2}$
36. $\log_b 2 = \frac{1}{2}$
37. $\log_{10} x = 0$
38. $\log_{10} x = 1$

Given that $\log_2 5 = 2.3219$ and $\log_2 7 = 2.8074$, evaluate each of the following by using Properties 5.5–5.7.

39. $\log_2 35$
40. $\log_2\left(\frac{7}{5}\right)$
41. $\log_2 125$
42. $\log_2 49$
43. $\log_2\sqrt{7}$
44. $\log_2\sqrt[3]{5}$
45. $\log_2 175$
46. $\log_2 56$
47. $\log_2 80$

Given that $\log_8 5 = 0.7740$ and $\log_8 11 = 1.1531$, evaluate each of the following using Properties 5.5–5.7.

48. $\log_8 55$
49. $\log_8\left(\frac{5}{11}\right)$
50. $\log_8 25$
51. $\log_8\sqrt{11}$
52. $\log_8(5)^{2/3}$
53. $\log_8 88$
54. $\log_8 320$
55. $\log_8\left(\frac{25}{11}\right)$
56. $\log_8\left(\frac{121}{25}\right)$

Express each of the following as the sum or difference of simpler logarithmic quantities. (Assume that all variables represent positive real numbers.) For example,

$$\log_b\left(\frac{x^3}{y^2}\right) = \log_b x^3 - \log_b y^2$$
$$= 3\log_b x - 2\log_b y.$$

57. $\log_b xyz$
58. $\log_b\left(\frac{x^2}{y}\right)$
59. $\log_b x^2y^3$
60. $\log_b x^{2/3}y^{3/4}$
61. $\log_b\sqrt{xy}$
62. $\log_b\sqrt[3]{x^2z}$
63. $\log_b\sqrt{\frac{x}{y}}$
64. $\log_b\left[x\left(\sqrt{\frac{x}{y}}\right)\right]$

Express each of the following as a single logarithm. (Assume that all variables represent positive numbers.) For example,

$$3\log_b x + 5\log_b y = \log_b x^3y^5.$$

65. $\log_b x + \log_b y - \log_b z$
66. $2\log_b x - 4\log_b y$

67. $(\log_b x - \log_b y) - \log_b z$

68. $\log_b x - (\log_b y - \log_b z)$

69. $\log_b x + \frac{1}{2}\log_b y$

70. $2 \log_b x + 4 \log_b y - 3 \log_b z$

71. $2 \log_b x + \frac{1}{2}\log_b(x - 1) - 4 \log_b(2x + 5)$

72. $\frac{1}{2}\log_b x - 3 \log_b x + 4 \log_b y$

Solve each of the following equations.

73. $\log_3 x + \log_3 4 = 2$

74. $\log_7 5 + \log_7 x = 1$

75. $\log_{10} x + \log_{10}(x - 21) = 2$

76. $\log_{10} x + \log_{10}(x - 3) = 1$

77. $\log_2 x + \log_2(x - 3) = 2$

78. $\log_3 x + \log_3(x - 2) = 1$

79. $\log_{10}(2x - 1) - \log_{10}(x - 2) = 1$

80. $\log_{10}(9x - 2) = 1 + \log_{10}(x - 4)$

81. $\log_5(3x - 2) = 1 + \log_5(x - 4)$

82. $\log_6 x + \log_6(x + 5) = 2$

83. $\log_8(x + 7) + \log_8 x = 1$

84. $\log_6(x + 1) + \log_6(x - 4) = 2$

85. Verify Property 5.6.

86. Verify Property 5.7.

THOUGHTS into WORDS

87. How would you explain the concept of a logarithm to someone who has never studied algebra?

88. Explain, without using Property 5.4, why $4^{\log_4 9}$ equals 9.

89. In the next section we are going to show that the logarithmic function $f(x) = \log_2 x$ is the inverse of the exponential function $f(x) = 2^x$. From that information how could you sketch a graph of $f(x) = \log_2 x$?

5.4 LOGARITHMIC FUNCTIONS

The concept of a logarithm can now be used to define a logarithmic function.

DEFINITION 5.3

If $b > 0$ and $b \neq 1$, then the function defined by

$$f(x) = \log_b x,$$

where x is any positive real number, is called the **logarithmic function with base b**.